10. cvičení - výsledky

15. 12. 2022

Příklad 1

(a) $\mathcal{D}_f = [0,\infty)$; $f'(x) = \frac{\sqrt{3}}{2\sqrt{x}} - 51x^2 + 5x^4$; $\mathcal{D}_{f'} = (0,\infty)$

(b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = e^x(1+x^2)$; $\mathcal{D}_{f'} = \mathbb{R}$

(c) $\mathcal{D}_f = [0,\infty)$; $f'(x) = 1 + \frac{1}{4x^{3/4}} + \frac{1}{\sqrt{x}}$; $\mathcal{D}_{f'} = (0,\infty)$

(d) $\mathcal{D}_f = (0,\infty)$; $f'(x) = \log(x) + 1 + \log_3(x) + \frac{1}{\log(3)} + 2e^x$; $\mathcal{D}_{f'} = (0,\infty)$

(e) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \log(\frac{3}{2})(\frac{3}{2})^x$; $\mathcal{D}_{f'} = \mathbb{R}$

(f) $\mathcal{D}_f = \mathbb{R}\setminus\{0\}$; $f'(x) = \frac{9-4x-x^2}{x^4}$; $\mathcal{D}_{f'} = \mathbb{R}\setminus\{0\}$

(g) $\mathcal{D}_f = \mathbb{R}\setminus\{-6,1\}$; $f'(x) = \frac{-23-26x+7x^2}{(x-1)^2(6+x)^2)}$; $\mathcal{D}_{f'} = \mathbb{R}\setminus\{-6,1\}$

(h) $\mathcal{D}_f = (0,\infty)$; $f'(x) = \frac{1}{x} - \frac{\sin x}{\pi}$; $\mathcal{D}_{f'} = (0,\infty)$

(i) $\mathcal{D}_f = (0,\infty)\setminus\{1\}$; $f'(x) = \frac{2+x\cos(x)(-1+\log(x))-x^2\log(x)\sin(x)}{x\log^2(x)}$; $\mathcal{D}_{f'} = (0,\infty)\setminus\{1\}$

(j) $\mathcal{D}_f = \mathbb{R}\setminus(\pi/2+\pi\mathbb{Z})$; $f'(x) = \frac{1}{\cos^2(x)} - \cos(x)$; $\mathcal{D}_{f'} = \mathcal{D}_f$

(k) $\mathcal{D}_f = \mathbb{R}\setminus(\pi/2+\pi\mathbb{Z})$; $f'(x) = \cos^2(x) - \sin^2(x) - \frac{1}{\cos^2(x)}$; $\mathcal{D}_{f'} = \mathcal{D}_f$

(l) $\mathcal{D}_f = [-1,1]$; $f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{3}{(1+x^2)}$; $\mathcal{D}_{f'} = (-1,1)$

(m) $\mathcal{D}_f = [-1,1]$; $f'(x) = -\frac{5}{\sqrt{1-x^2}} + \frac{2}{1+x^2}$; $\mathcal{D}_{f'} = (-1,1)$

Příklad 2

(a) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = 78(x^2+50x+12)^{77}\cdot(x^2+50x+12)$; $\mathcal{D}_{f'} = \mathbb{R}$

(b) $\mathcal{D}_f = \mathbb{R}\setminus\{7\}$; $f'(x) = -\frac{3x^2(x+2)^7(31x+14)}{(x-7)^{-12}}$; $\mathcal{D}_{f'} = \mathbb{R}\setminus\{7\}$

(c) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = -\frac{1}{1+4x^2}\cdot 2$; $\mathcal{D}_{f'} = \mathbb{R}$

(d) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \frac{3+4x-3x^2}{(1+x^2)^2}$; $\mathcal{D}_{f'} = \mathbb{R}$

(e) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \frac{2x+1}{x^2+x+2}$; $\mathcal{D}_{f'} = \mathbb{R}$

(f) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = -\sin(x^3-x+2)^9\cdot 9(x^3-x+2)^8\cdot(3x^2-1)$; $\mathcal{D}_{f'} = \mathbb{R}$

(g) $\mathcal{D}_f = [-2,2]$; $f'(x) = -\frac{x}{\sqrt{4-x^2}}$; $\mathcal{D}_{f'} = (-2,2)$

(h) $\mathcal{D}_f = (-3,3)$; $f'(x) = 9\cdot(9-x^2)^{-\frac{3}{2}}$; $\mathcal{D}_{f'} = (-3,3)$

(i) $\mathcal{D}_f = \mathbb{R} \setminus \{-1\}$; $f'(x) = \frac{x(1-x)^2\left(2-4x^2-4x\right)}{(1+x)^2}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{-1\}$

(j) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = 2e^{-\frac{1}{x^2}} \frac{1}{x^3}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$

(k) $\mathcal{D}_f = (0, \infty)$; $f'(x) = x^x \cdot (\log x + 1)$; $\mathcal{D}_{f'} = (0, \infty)$

(l) $\mathcal{D}_f = (0, \infty)$; $f'(x) = \left(\frac{1}{x}\right)^{\frac{1}{x}} \cdot \left(\frac{-\log x}{x^2} + 1\right)$; $\mathcal{D}_{f'} = (0, \infty)$

(m) $\mathcal{D}_f = \bigcup_{k\in\mathbb{Z}} (2k\pi, \pi + 2k\pi)$; $f'(x) = e^{\cos x \cdot \log \sin x} \cdot \left(-\sin x \log \sin x + \frac{\cos^2 x}{\sin x}\right)$; $\mathcal{D}_{f'} = \bigcup_{k\in\mathbb{Z}} (2k\pi, \pi + 2k\pi)$

(n) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \frac{-\sin x}{\sqrt{1-\cos^2 x}}$; $\mathcal{D}_{f'} = \mathbb{R}$

(o) $\mathcal{D}_f = [-1, 1)$; $f'(x) = \frac{1}{\arccos x} \cdot \frac{-1}{\sqrt{1-x^2}}$; $\mathcal{D}_{f'} = (-1, 1)$

(p) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = \frac{-1}{2+x^2}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$

(q) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = e^{x^2-x+1} \cdot (2x + 1) - \sqrt{\frac{e^{2x}+1}{e^{2x}}} \cdot \frac{e^{2x}\left(1-e^{2x}\right)}{\left(e^{2x}+1\right)^2}$; $\mathcal{D}_{f'} = \mathbb{R}$

(r) $\mathcal{D}_f = \mathbb{R} \setminus (0, 1)$; $f'(x) = \frac{1}{x\sqrt{x^2-1}} - \frac{\sqrt{x^2-1} - \log x \cdot 2x^2}{x^3-x}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus (0, 1)$

(s) $\mathcal{D}_f = [-1, 1]$; $f'(x) = \left(\arcsin\left(x^3\right)\right)^2 + 2x \cdot \arcsin\left(x^3\right) \cdot \frac{3x^2}{\sqrt{1-x^6}}$; $\mathcal{D}_{f'} = (-1, 1)$

Příklad 3

(a) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = f'(x) = \begin{cases} -1, & x \in (-\infty, 0) \\ 1, & x \in (0, \infty). \end{cases}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$; v $x = 0$ je derivace zleva -1, zprava 1

(b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = f' = \begin{cases} 3 - 2x, & x \in (-\infty, 0) \cup (3/2, \infty), \\ 2x, & x \in (0, 3/2) \end{cases}$,
$f'_-(0) = 3, f'_+(0) = 0, f'_-(3/2) = 3, f'_+(3/2) = 0$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0, \frac{3}{2}\}$

(c) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = f' = \begin{cases} 1/x, & x \in (-\infty, -1) \cup (1, \infty), \\ -1/x, & x \in (-1, 0) \cup (0, 1), \end{cases}$,
$f'_-(-1) = -1, f'_+(-1) = 1, f'_-(0) = \infty, f'_+(0) = -\infty, f'_-(1) = -1, f'_+(1) = 1$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{-1, 0, 1\}$

(d) $f'(x) = \operatorname{sgn} x$ na $\mathbb{R} \setminus \{0\}$

(e) $f' = \begin{cases} 1, & x \in (-\infty, 0], \\ 1/(1 + x), & x \in (0, \infty) \end{cases}$,
Pozor. Bod $x = 0$ je třeba vyšetřit zvláště.

Příklad 4

(a) $\mathcal{D}_f = (0, \infty) \setminus \{1\}$; $f'(x) = -\frac{1}{x \log^2 x}$; $\mathcal{D}_{f'} = (0, \infty) \setminus \{1\}$

(b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = 9\cos^8(x^3 - x + 2) \cdot (-\sin(x^3 - x + 2) \cdot (3x^2 - 1)$; $\mathcal{D}_{f'} = \mathbb{R}$

(c) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \cos(\sin(\sin(x))) \cdot \cos(\sin(x)) \cdot \cos x$; $\mathcal{D}_{f'} = \mathbb{R}$

(d) $\mathcal{D}_f = (-\infty, 1) \cup (1, \infty)$; $f'(x) = \frac{8x}{x^4-1}$; $\mathcal{D}_{f'} = (-\infty, 1) \cup (1, \infty)$

(e) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \sin(2x) - 2x\cos(x^2)$; $\mathcal{D}_{f'} = \mathbb{R}$

(f) $\mathcal{D}_f = (1, \infty) \setminus \{e\}$; $f'(x) = \frac{6}{x\log(x)\log(\log^3(x))}$; $\mathcal{D}_{f'} = (1, \infty) \setminus \{e\}$

(g) $f' = \begin{cases} 2, & x \in (-\infty, -3) \cup (1, \infty), \\ -2x, & x \in (-3, 1) \\ nedef, & x \in \{-3, 1\} \end{cases},$

$f'_-(-3) = 2$, $f'_+(-3) = 6$, $f'_-(1) = -2$, $f'_+(3/2) = 2$

(h) $f' = \begin{cases} -1, & x \in (-\infty, 1], \\ 2x - 3, & x \in (1, 2), \\ 1, & x \in [2, \infty) \end{cases},$

Pozor. Body $x = 1$ a $x = 2$ je třeba vyšetřit zvláště.

(i) $f' = \begin{cases} -\frac{1}{3}x^{2/3}\cos\frac{1}{\sqrt[3]{x}} + 2x\sin\frac{1}{\sqrt[3]{x}}, & x \neq 0, \\ 0, & x = 0 \end{cases},$

Pozor. Bod $x = 0$ je třeba vyšetřit zvláště.

(j) $f' = \begin{cases} 0, & x \neq 0, \\ -\infty, & x = 0 \end{cases}$